THE HONG KONG POLYTECHNIC UNIVERSITY

Department of Applied Mathematics

Lightboard Project

6. Higher Derivatives, Implicit Differentiation and Derivative of Inverse Function

- 6.1 Suppose $f(x) = 3x + \cos(3x) + e^x$. (a) Show that the inverse function f^{-1} exists. (b) Find $(f^{-1})'(2)$. [21222 Test2]
- 6.2 Assume that $y = f(x) = 2e^x \cos(x) + \arctan(2x)$ has an inverse function, denoted by f^{-1} . Find $f^{-1}(1)$ and $(f^{-1})'(1)$. [22231 Test2]
- 6.3 If the equation $y \ln(2x + y) = e^x + 2y 3$ defines a differentiable function y = h(x) in a suitable region of the xy coordinate system, evaluate the derivative $\frac{dy}{dx}$ at the point (x, y) = (0, 1) [18192 Test2]
- 6.4 Suppose the equation $\arctan(x+2y+1) = y^5$ defines a differentiable function y = f(x), find $\frac{dy}{dx}$ and evaluate it at (-1, 0). [21222 Exam]
- 6.5 If the equation $e^x + e^y = x^e + y^e + 2e^2 2(2^e)$ defines a differentiable function y = f(x), find and evaluate the derivative $\frac{dy}{dx}$ at the point (2,2).

[15161 Exam]

- 6.6 Suppose the equation $\cos(xy) = \frac{2^y}{x^3}$ defines a differentiable function y = f(x), find $\frac{dy}{dx}$ and evaluate it at (1,0). [17182 Exam]
- 6.7 Consider the function $f(x) = x^2 g(x)$, where g(x) is twice differentiable and g(1) = 2, g'(1) = -1 and g''(1) = 3. Find f''(1). [18192 Test2]

6.8 A function f and its first and second order derivatives f' and f'' are shown in the following table. If $g(x) = e^{2x} f(x^2 - x)$, find the values of its first and second derivatives g' and g'' for x = 0 and 1.

	f(x)	f'(x)	f''(x)
x = 0	1	2	3
x = 1	4	5	6

[14152 Exam]

6.9 Find the n^{th} order derivative of the following function:

$$y = \ln x$$
.

6.10 Given that $x = \frac{1}{1+t^2}$, $y = \frac{t^3}{1+t^2}$, obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t.